Multifrequency measurements of core-diffracted $P$ waves (Pdiff) for global waveform tomography

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SUMMARY
The lower third of the mantle is sampled extensively by body waves that diffract around the earth’s core (Pdiff and Sdiff phases), which could deliver highly resolved tomographic images of this poorly understood region. But core-diffracted waves—especially Pdiff waves—are not often used in tomography because they are difficult to model adequately. Our aim is to make core-diffracted body waves usable for global waveform tomography, across their entire frequency range. Here we present the data processing part of this effort. A method is demonstrated that routinely calculates finite-frequency traveltimes of Pdiff waves by cross-correlating large quantities of waveform data with synthetic seismograms, in frequency passbands ranging from 30.0 to 2.7 s dominant period. Green’s functions for 1857 earthquakes, typically comprising thousands of seismograms, are calculated by theoretically exact wave propagation through a spherically symmetric earth model, up to 1 Hz dominant period. Out of 418 226 candidates, 165 651 (39.6 per cent) source–receiver pairs yielded at least one successful passband measurement of a Pdiff traveltime anomaly, for a total of 479 559 traveltimes in the eight passbands considered. Measurements of teleseismic $P$ waves yielded 448 178 usable source–receiver paths from 613 057 candidates (73.1 per cent success rate), for a total of 2 306 755 usable teleseismic $dT$ in eight passbands. Observed and predicted characteristics of Pdiff traveltimes are discussed and compared to teleseismic $P$ for this very large data set. Pdiff measurements are noise-limited due to severe wave attenuation with epicentral distance and frequency. Measurement success drops from 40–60 per cent at $80^\circ$ distance, to 5–10 per cent at $140^\circ$. Frequency has a 2–3 times stronger influence on measurement success for Pdiff than for $P$. The fewest usable $dT$ measurements are obtained in the microseismic noise band, whereas the fewest usable teleseismic $P$ measurements occur at the highest frequencies. $dT$ anomalies are larger for Pdiff than for $P$, and frequency dependence of $dT$ due to 3-D heterogeneity (rather than just diffraction) is larger for Pdiff as well. Projecting the Pdiff traveltime anomalies on their core-grazing segments, we retrieve well-known, large-scale structural heterogeneities of the lowermost mantle, such as the two Large Low Shear Velocity Provinces, an Ultra-Low Velocity Zone west of Hawaii, and subducted slab accumulations under East Asia and Central America.

Key words: Time-series analysis; Numerical solutions; Body waves; Seismic tomography; Wave scattering and diffraction; Wave propagation.

1 INTRODUCTION
Core-diffracted waves are seismic body waves that dive deep enough to sense the earth’s core, and by interaction with this boundary become dispersive. In ray-theoretical modelling, the transition from teleseismic to core-diffracted wave occurs at the epicentral distance where the deepest ray segment no longer turns in the mantle, but starts to graze the core–mantle boundary (CMB). For $P$ waves, this happens at $\approx 98^\circ$ epicentral distance if source and receiver are located near the surface. In reality, ray theory is a poor approximation of the true, finite-frequency sensitivity of a core-diffracted wave, which resembles a spatially extended banana in the mantle, but flattens elliptically on top of the CMB due to diffraction (Liu & Tromp 2008). Actual body waves already sense the core when the banana’s lower lobe extends to the CMB, which happens at significantly smaller distances than $98^\circ$.

Core-diffracted waves, and Pdiff waves in particular, have been used to study lowermost mantle structure (Su & Dziewonski 1997, ...
Masters et al. 2000; Ritsema & van Heijst 2002; Li et al. 2008),
but they have seen very little use in tomography, despite being
the most highly resolving wave type that extensively samples the
lower third of the mantle. The problem is that modelling their wave propa-
gation and non-ray-like sensitivities to a reasonable approximation
is challenging, requiring computationally expensive forward mod-
elling techniques. Global P-wave tomographies often include data
in the far teleseismic range, which effectively start to sense the
core (generally neglected by the sensitivity modelling), but they
stop short of including ‘real’ Pdiff data. Sparser sampling and
poorer modelling result in coarser image resolution and signif-
ificant discrepancies across tomographic models, even for large-scale
features. Hence, our knowledge of structural detail in the lower-
most mantle is lacking compared to shallower levels. For example,
many regional and global tomographies have proposed detailed tec-
tonic or paleogeographic interpretations of slab geometries imaged
at upper and mid mantle depths (e.g. for Tethyan slabs: Van der
Voo et al. 1999b); Hafkenscheid et al. (2006); for North America:
Grand (2002); Ren et al. (2007); Pavlis et al. (2012); Sigloch &
Mihalynuk (2013); globally: Li et al. (2008); van der Meer et al.
(2010)). The same has rarely been attempted for slabs in the lower
third of the mantle (Van der Voo et al. 1999a; van der Meer et al.
2010), despite agreement between virtually all global tomography
models that extensive fast-velocity provinces are present at these
defths.

To our knowledge, the only global P-wave tomographies to ex-
plitly model and include Pdiff travelt ime data were by Wysession
(1996) and Kára son & Van der Hilst (2001), both using a set of 543
the mantle path by ray theory and sensitivity at the CMB by an ap-
proximate elliptical Fresnel zone of constant value. Kára son & Van
der Hilst (2001) improved on this by constructing approximate 3-D
sensitivity kernels instead of assuming a constant Fresnel zone, and
inverted for 3-D mantle structure rather than a 2-D map of velocity
anomalies on the CMB. Resolution was obviously limited with only
543 measurements (compared to half a million presented here).
The same kernels and data set were reused by Li et al. (2008).

A few global S-wave tomographies explicitly include Sdiff data.
Ritsema et al. (2004, 2011) include cross-correlation travelt ime
measurements of SH-diff waves and model them by ray-theoretical
sensitivities. Még n & Romanowicz (2000) and Panning &
Romanowicz (2006) include SH-diff data in their waveform in-
version, modelling sensitivities by mode coupling in the NACT ap-
proximation, which features spatially extended sensitivities in the
source–receiver plane, but no sensitivity perpendicular to it. Wave
periods in these models were limited to 30 s and longer.

Most global tomographies include non-diffracted wave types
that sample the lowermost mantle, for example, core phases, core-
reflected phases, long-range teleseismic waves near the diffraction
limit, or normal modes. The structural findings from these data
have converged robustly on certain very large scale features in the
lower mantle, such as Large Low Shear Velocity Provinces (e.g.
Woodhouse & Dziewonski 1989; Masters et al. 2000; Még n &
Romanowicz 2000; Grand 2002; Ritsema & van Heijst 2002; Mon-
telli et al. 2006), but resolution remains limited by the long wave-
lengths used and/or by modelling mismatches inherent in using
approximate sensitivities.

We aim to improve on the state of the art as follows:

(1) Model and measure core-diffracted body waves across their
entire spectrum, up to the highest occurring frequencies. The goal
is maximum spatial resolution.

(2) ‘No data left behind’: a processing strategy efficient enough
to assemble the largest possible data sets (Hosseini-zad et al. 2012).
(3) Use of Pdiff in addition to Sdiff.
(4) Better approximation of true wave sensitivities by 3-D Born-
Fréchet kernels.

Items 1 and 2 aim for maximum spatial resolution and coverage
globally. In terms of wave type, we started with Pdiff because it
will integrate seamlessly with our teleseismic P-wave inversions,
but the processing methods presented here carry over to Sdiff with
only minimal changes. For the ambitious data volumes, frequen-
cies and distances targeted, fully numerical modelling in global 3-D
reference models (e.g. Komatitsch & Tromp 2002a) remains well
beyond reach, but the semi-analytical modelling of 3-D wave fields
and sensitivities in spherically symmetric earth models is becom-
ing feasible. We discuss the forward modelling of core-diffracted
seismograms, deferring the calculation of Born-Frédéc ket sensitiv-
ity kernels to a later paper.

Section 2.1 introduces the observational characteristics of Pdiff
waves: strong dispersion and attenuation as functions of fre-
quency and epicentral distance. Section 2.2 describes our successful
adoption of two recent numerical packages for computing broad-
band Green’s functions: Yspec (Al-Attar & Woodhouse 2008) and
AxisSEM (Nissen-Meyer et al. 2014). In practice, the success of fit-
ting broad-band waveforms depends not only on accurate Green’s
functions but equally on reliable estimates of source time functions
and source depths. We compute traveltime measurements by cross-
correlation in multiple frequency bands—essentially frequency-
dependent phase shifts. Section 3 analyses the global data set of
multifrequency traveltimes obtained so far: 479 559 P-diffracted
and 2 306 755 teleseismic P measurements. We discuss how me-
surement success depends on epicentral distance, frequency band
and earthquake magnitude. The statistics of traveltimes for Pdiff
and teleseismic P are presented, including their frequency depen-
dence. The information content of the novel Pdiff measurements
is demonstrated by projecting traveltime anomalies on the core-
grazing segments of their nominal ray paths. Already from this
‘proto-tomography’ exercise, structural heterogeneity of consider-
able detail emerges, which in Section 4 is compared to current
structural knowledge about the lowermost mantle.

2 WAVEFORM DATA AND PROCESSING

2.1 Waveform recordings of core-diffracted P waves

We start by discussing the nature of P-diffracted waves. As the
travel distance of a teleseismic P wave increases and transitions
into the core-diffracted regime, the character of the waveform changes.
Fig. 1 illustrates this for broad-band data from a deep earthquake of
magnitude 7.5 in Southern Sumatra (2009/09/30 10:16:09, 0.72 S,
99.87 E, depth: 82.0 km). Sharply defined pulses recorded in the
teleseismic distance range (Δ < 90 °) morph into increasingly emer-
gent oscillations at distances exceeding 100°. This is clearly ob-
servable for the isolated (blue) P pulse arrival in the real data
(Fig. 1a), and even more evident in the modelled Green’s func-
tions of Fig. 1(c). (Broad-band Green’s functions were computed
with the Yspec software of Al-Attar & Woodhouse (2008), described
in Section 2.2.) At ranges Δ < 100°, two surface phases pP and
sP (in red) are clearly distinguished 20–40 s after the P arrival,
whereas in the Pdiff range these two pulses are smeared together.
In other words, high-frequency content is lost disproportionally with
distance in the core-diffracted range, while this is not the case in the teleseismic range. The increasingly emergent and low passed character of the Pdiff pulses in Figs 1(a)–(c) indicates that ray-theoretical processing methods such as manual or automated onset picking would be inadequate because they rely on sharp, impulsive onsets. The emergent onsets are a manifestation of the different physics of diffracted wave propagation, for which the ray-theoretical approximation breaks down.

The dependence of wave amplitude on source–receiver distance is not apparent in Fig. 1 because each trace is energy-normalized, but it differs fundamentally between P and Pdiff (Knopoff & Gilbert 1961; Sacks 1966). In a computational experiment, we calculated broad-band Green’s functions as in Fig. 1(c), except that an explosive source was used, 100 km deep in order to cleanly isolate the P pulse from its echoes. The vertical component waveforms were filtered to eight overlapping frequency passbands with dominant periods ranging from 30.0 to 2.7 s. (The filters are used throughout this study and described in Section 2.3.2.) RMS amplitudes of the bandpassed Green’s functions were measured at epicentral distances from 40° to 150° in increments of 1°.

Fig. 2 shows the result: with increasing travel distance, amplitudes drop much more severely for Pdiff than for P. This is true in any frequency band, but particularly pronounced for high frequencies. Straight lines in this lin-log plot indicate exponential loss of energy with distance, the negative line slope being the attenuation constant in the exponent. Slopes in the P-diffracted range are steep and strongly dispersive—in the far Pdiff range, higher frequencies can be suppressed by orders of magnitude. This creates a signal-to-noise challenge for Pdiff measurements, and Section 3.1 investigates its practical consequences.

The Pdiff slopes of Fig. 2 are found to be in good agreement with analytical predictions of (Aki & Richards 2002, p. 457), superimposed as dash-dotted lines. They predict the amplitude A of core-diffracted P waves to be proportional to

\[
A \propto \exp\left[-\omega^{1/3}\lambda \sin(\pi/3)(\Delta_0 - \Delta_d)\right],
\]

where \(\omega\) is the angular frequency, \(\lambda\) is a positive constant and \(\Delta_0\) is the source–receiver distance. In the teleseismic range, the P and PcP pulses approach each other in time with increasing \(\Delta_0\) until they merge at \(\Delta_d\), which is called the shadow boundary. Beyond \(\Delta_d\), the rays start to creep around the CMB as diffracted P waves. With increasing frequency and/or distance (both in the exponent), the wave amplitude of Pdiff attenuates rapidly. To create the dash-dotted lines in Fig. 2, we estimated the constant \(\lambda\) by fitting above analytical solution to our results for the highest frequency band (2.7 s). This \(\lambda\) value was then used to calculate slopes for the other two frequency bands (5.3 s and 10.6 s), and the result is very consistent with the prediction.

Figure 1. Observed versus modelled broad-band seismograms of teleseismic and core-diffracted P waves. Record section of a deep magnitude 7.5 earthquake in Southern Sumatra (2009/09/30 10:16:09.2, at 0.72°S, 99.87°E, 82 km depth), recorded on seismic networks globally. (a) Observed broad-band waveforms aligned on their ray-theoretical P-arrival times. Each line shows a seismogram (vertical component) of normalized RMS amplitude, from 20 s before to 50 s after P arrival; positive samples are blue, negative samples are red. Waveforms include the direct P phase (blue), and in its wake the Pp and pP surface reflections (red). The Pdiff epicentral distance range >120° is particularly densely sampled by several hundred USArray stations. The increasingly broad and emergent pulses beyond distances of 100° result from diffraction around the earth’s core. Red samples preceding the P arrival are a filter response resulting from highpass filtering at 100 s corner period, which serves to suppress low-frequency noise. (b) Broad-band synthetics, the convolution of the Green’s functions in panel (c) with the source time function in panel (d). (c) Broad-band Green’s functions (up to 1 Hz dominant period) computed in a spherically symmetric earth model by the Yspec method of Al-Attar & Woodhouse (2008). (d) The broad-band source time function (amplitude normalized), which was deconvoluted from panels (a) and (c).
The second dashed line, around 98° at a significantly smaller distance than predicted by ray theory. A reference earth model convolved with an estimate of the earth’s Green’s function (Fig. 1c) with a source time function estimate (Fig. 1d). Amplitudes $A$ in each band are normalized to the amplitude $A_w$ of a $P$ wave at 40° distance. In this lin-log plot, straight lines indicate exponential loss of wave amplitude with distance, and negative line slopes represent the attenuation constants in the exponent. Divergence of coloured lines in the Pdiff range indicates frequency dispersion. Dash-dotted slopes are ray-theoretical predictions (Aki & Richards 2002, p. 457), vertical dashed lines predict the distances at which a ray-theoretical $P$ wave would first hit the CMB, respectively.

Figure 2. $P$-diffracted waves are highly attenuative and dispersive compared to teleseismic $P$ waves—the effect of diffraction around the earth’s core. As a function of epicentral distance, the logarithmic RMS amplitudes $\ln (A/A_0)$ of Green’s functions are plotted. Explosive source, 100 km deep, vertical receiver component, computed up to 1 s dominant period in velocity model IASP91 including PREM attenuation.) Coloured lines distinguish eight frequency passbands, labelled by their dominant periods. Amplitudes $A$ in each band are normalized to the amplitude $A_w$ of a $P$ wave at 40° distance. In this lin-log plot, straight lines indicate exponential loss of wave amplitude with distance, and negative line slopes represent the attenuation constants in the exponent. Divergence of coloured lines in the Pdiff range indicates frequency dispersion. Dash-dotted slopes are ray-theoretical predictions (Aki & Richards 2002, p. 457), vertical dashed lines predict the distances at which a ray-theoretical $P$ wave would first hit the $D''$ and the CMB, respectively.

The transition from the $P$ to the Pdiff regime in Fig. 2 occurs at a significantly smaller distance than predicted by ray theory. The second dashed line, around 98°, marks the predicted onset of core-grazing rays for the deep source depth used (100 km depth). However, waves in our physically more realistic simulation have volumetrically extended, banana-like sensitivities to mantle structure, and start to sense the core at distances as small as 85°. ‘Teleseismic’ waves at those distances are not well modelled by ray theory.

2.2 Modelling core-diffracted waves

Broad-band waveform tomography on a global scale has three modelling requirements:

1. Green’s functions;
2. source time functions;
3. sensitivity kernels.

The main obstacle to exploiting core-diffracted waves has been modelling them to a sufficiently realistic degree. We employ recently developed, semi-analytical tools for forward wave propagation and discuss the computation of Green’s functions and source time functions. Sensitivity kernels are required for inversion of the data in a linearized optimization problem. (Unless global search approaches are used, which only rely on a large number of forward calculations but are computationally out of reach for the number of parameters required in 3-D inversions.) Tomographic inversion of our Pdiff data will be the topic of a follow-up paper, but we briefly discuss how the numerical tools for Green’s function and kernel computations relate.

An observed seismogram is the convolution of an earthquake’s source time function with the earth’s Green’s function between a given source–receiver pair. Waveform tomography compares this to a predicted seismogram, which is a Green’s function computed in a reference earth model convolved with an estimate of the earthquake’s source time function. Fig. 1 shows a typical broad-band data example for observed seismograms (Fig. 1a) and their predicted counterparts (Fig. 1b), which in turn are the convolution of the computed Green’s functions (Fig. 1c) with a source time function estimate (Fig. 1d).

2.3 Green’s functions

We compute Green’s functions by simulating wave propagation in a broad-banded spectrum from 0.2 mHz to 1 Hz in the spherically symmetric earth model IASP91 (Kennett & Engdahl 1991) with density and attenuation from PREM (Dziewonski & Anderson 1981).

Two numerical software packages are suitable for these calculations: Yspec (Al-Attar & Woodhouse 2008) and AxiSEM (Nissen-Meyer et al. 2007, 2014). Yspec is very efficient for calculating synthetic seismograms in spherically symmetric earth models using the direct radial integration method (Woodhouse 1980; Friederich & Dalkolmo 1995), which can account for the full physics of such media including viscoelastic damping (used here) and transverse isotropy (not used here). AxiSEM is a spectral-element code that computes 3-D global seismic wavefields for full moment tensor sources in viscoelastic (van Driel & Nissen-Meyer 2014a), anisotropic (van Driel & Nissen-Meyer 2014b) media across the observable frequency band at a reasonable computational cost.

Both Yspec and AxiSEM are semi-analytical methods that solve the full physics of wave propagation, but make use of the earth model’s assumed spherical symmetry to analytically reduce the cost of computing 3-D wavefields—as compared to fully numerical simulations of wave propagation in 3-D heterogeneous earth models (e.g. Komatitsch & Tromp 2002a; Fichtner et al. 2009). Aside from numerical imprecisions, either approach computes 3-D wavefields that are theoretically exact within its modelling assumptions, but only the semi-analytical computations are efficient enough for our application.
Yspec and AxiSEM have been benchmarked against each other between 1 mHz and 1 Hz, yielding virtually indistinguishable seismograms (Nissen-Meyer et al. 2014). Since the implementation of attenuation and anisotropy in AxiSEM were not yet complete, all traces in this study were calculated with Yspec, which was very efficient for our forward modelling purposes. An example of its broad-band Green’s functions is shown in Fig. 1(c). Yspec took \( \approx 480 \) CPU hours for our typical event of \( \approx 3000 \) seismograms (1000 stations, 3 components), up to a dominant frequency of 1 Hz. This is an order of magnitude faster than AxiSEM for a single event. Computation scaled linearly with the number of sources, and we computed 2000 earthquakes for this study, expending a total of \( \approx 10^6 \) CPU hours. Sensitivity kernels are much more expensive to compute than Green’s functions. For our present application, only AxiSEM is suitable for sensitivity kernel calculations thanks to its efficiency in computing and storing the spatiotemporal evolution of the 3-D wavefields. Hence, for the inversion stage we will switch to AxiSEM.

AxiSEM and Yspec are semi-analytical, theoretically exact methods that include the full physics of wave propagation but exploit (spherical) symmetry in the background model for efficient computation. By contrast, our earlier studies used asymptotic (and therefore approximate) forward modelling methods, also assuming spherical symmetry. For teleseismic P measurements, Sigloch & Nolet (2006) used the WKBJ method of Chapman (1978); and for triplicated body waves, Stähler et al. (2012) used the reflectivity method by Fuchs & Müller (1971). Fig. 3 compares broad-band waveforms, WKBJ synthetics and Yspec synthetics for the Sumatra event of Fig. 1. While agreeing in their basic characteristics, there are clear differences between WKBJ and Yspec synthetics in the teleseismic range, where both are applicable. (WKBJ does not compute core-diffracted waves.) Differences between AxiSEM and Yspec are much smaller (Nissen-Meyer et al. 2014) than the difference between Yspec and the asymptotic WKBJ method.

2.3.2 Multifrequency cross-correlation traveltimes for tomography

The measurement procedure for multifrequency traveltimes is again based on that for teleseismic body waves by Sigloch & Nolet (2006). A brief summary follows, highlighting Pdiff-related adaptations.

(1) Processing proceeds earthquake by earthquake, typically on events exceeding magnitude \( m_s > 5.5 \). A pre-requisite is an estimate of the broad-band source time function (Section 2.3.1).

(2) A predicted broad-band seismogram or matched filter for a receiver \( r \) at an arbitrary distance range is computed by convolving its broad-band Green’s function (from Yspec or AxiSEM) with the event’s broad-band source time function.

(3) Broad-band observed and predicted seismograms are passed through the same filter bank of bandpass filters. We use Gabor filters in eight overlapping frequency bands, with dominant periods ranging between 30.0 and 2.7 s (Fig. 4a).

(4) The cross-correlation function between bandpassed observed and predicted waveforms is computed in each passband. The time shift that maximizes this function is defined to be the finite-frequency traveltime anomaly \( dT_{s,b} \) (Dahlen et al. 2000). The tomographic inversion will use these traveltime anomalies as its measure of misfit, to be minimized in a least-squares sense. \( dT_{s,b} \) from all wave paths and frequency bands are required to jointly fit the (not frequency dependent) 3-D earth model solution.

Gabor filters are Gaussian functions in the log-frequency domain of constant fractional bandwidth and good spectral concentration characteristics. We space adjacent centre frequencies by \( \sqrt{2} \), see Sigloch & Nolet (2006) for filter details. Figs 4(b)–(d) demonstrate
Figure 4. Procedure for multifrequency measurements of traveltimes. (a) Frequency responses of the eight bandpass filters used, Gabor filters of constant fractional bandwidth. Centre periods range from 30.0 to 2.7 s (0.033 to 0.37 Hz) and are spaced by factors of $\sqrt{2}$. (b) Selected observations (red) and Yspec synthetics (black) for the Sumatra event, filtered to the lowest frequency band, that is, 30.0 s dominant period. Dashed line marks 1.5 times the duration of each filter’s impulse response, which we chose as the length of the correlation time window. The finite-frequency traveltime anomaly is defined as the time delay that maximizes the cross-correlation of observed and predicted waveforms within this time window. (c) Same as (b), but filtered to 15 s dominant period (band 3). (d) Same as (b), but filtered to 7.5 s dominant period (band 5).

As a misfit measure for tomography, cross-correlation traveltimes have very favourable properties. They are robust in that cross-correlation or matched filtering is the optimal way of detecting a signal of known shape in white Gaussian noise (Sigloch & Nolet 2006, and signal processing references therein). This robustness is important in environments of high ambient noise, such as Pdiff waves at long distance ranges and high frequencies. Cross-correlation traveltimes also retain a near-linear relationship to velocity model perturbations over a larger perturbation range than direct waveform differences (samplewise $L_2$ misfit) (Mercerat & Nolet 2013).

2.4 Pdiff wave dispersion caused by mantle heterogeneities

Interaction with the CMB makes P-diffracted waves dispersive in a spherically symmetric earth, in contrast to teleseismic P waves. However, in a planet with lateral velocity variations, $dT$ observations of teleseismic P waves also become dispersive, provided their wave lengths are of the same length scale as the mantle heterogeneity they interact with (Nolet & Dahlen 2000). Dispersion observed on Pdiff traveltimes will be a superposition of the two effects: diffraction around the core and finite-frequency effects caused by a 3-D heterogeneous earth.

Fig. 5 demonstrates the dispersion that arises from different spherically symmetric reference models. Besides the IASP91 model used throughout this study, we considered two models with differing velocity gradients in the D'' layer (2740–2899 km). The positive gradient model (blue in Fig. 5a) results in a P-wave velocity at the CMB that is 2.8 per cent faster than in IASP91. The negative gradient model (red) produces a 2.8 per cent slower $v_p$ at the CMB. The absolute gradient values were chosen as plausible regional variations from the global average of IASP91, following Thorne et al. (2013), who stated 2.8 per cent as characteristic S-velocity variations in D'' (but tomographic models tend to underestimate $dv/v$, hence we adopted the full 2.8 per cent for $dv_p/v_p$).

As expected, the fast gradient produces early arrival times compared to IASP91 (blue versus black waveforms in Fig. 5b), and the negative gradient produces delayed arrivals (red versus black waveforms). The advance and delay are 2.2 s and −2.3 s in the lowest frequency band (30 s period), and larger (3–4 s) in the higher frequency bands—this is traveltime dispersion caused by diffraction around the symmetric core. An actually observed Pdiff waveform
Figure 5. Observed and predicted traveltime dispersions of Pdiff, for different P-velocity models of the lowermost mantle. (a) P-velocity velocities in the D" layer region according to spherically symmetric reference model IASP91 (black), and two perturbed models featuring linear gradients that result in 2.8 per cent faster velocities at the CMB than IASP91 (blue), or 2.8 per cent slower velocities (red). (b) Observed (dashed) and predicted Pdiff seismograms for a wave path from the example Sumatra event to a station at 131.7° epicentral distance (USArray station J23A, shown in Fig. 6a. This is the blue wave path in Fig. 7a). Yspec synthetics were calculated through IASP91 and the two alternative models, and are shown for frequency passbands 1 (dominant period 30.0 s), 3 (15.0 s) and 5 (7.5 s). Traveltime delays $\Delta T = T^D - T^{D*}$ of the observed (dashed) seismograms relative to the synthetics, as computed through the fast model (blue), IASP91 (black), and the slow model (red). A single floating time shift $\Delta t_0$ between data and synthetics, which is due to uncertainties in absolute earthquake timing, has been arbitrarily fixed such that $\Delta t_{IASP91} = 0$ s in the lowest frequency band, that is, the observed arrival coincides with the IASP91-predicted arrival in this band (which facilitates visual comparison of the waveforms). The observed $\Delta T$ is dispersive: to higher frequencies ($T = 30, 15$ and $7.5$ s) the dashed waveform arrives increasingly early ($\Delta T = 0.0, -1.3$ and $-1.9$ s) w.r.t. IASP91. Comparison to the alternative models suggests a seismically fast wave path—the blue velocity profile produces dispersion of the same sign (increasingly advanced relative to IASP91), although of insufficient magnitude to explain this particular observation. See text for further discussion.

(grey dashed, from the example $M_w = 7.5$ Sumatra event to USArray station J23A, cf. map in Fig. 6a) shows the same sign of dispersion as the positive gradient model: increasingly early arrivals as frequency increases. This suggests an anomalously fast wave path in the lowermost mantle, for which IASP91 would be too slow a model—a plausible scenario for a core-grazing path beneath the northern Pacific (the blue path in Fig. 7a), because tomographic models generally infer an anomalously fast lowermost mantle beneath this region (e.g. Woodhouse & Dziewonski 1989; Masters et al. 2000; Mégnin & Romanowicz 2000; Grand 2002; Montelli et al. 2006; Ritsema et al. 2011). The fast anomalies seem to be quite strong judging by the observed dispersion: with increasing frequency, the relative $\Delta T$ advance of the observed seismogram (dashed) w.r.t. to IASP91 is larger than for the fast gradient model (blue waveforms) w.r.t. IASP91.

Fig. 6 investigates the magnitude of Pdiff traveltime dispersion that cannot be explained by spherically symmetric structure and must be due to 3-D heterogeneity, using a cluster of stations in the vicinity of station J23A just discussed. Traveltime measurement results of our processing scheme are shown for a set of 30 spatially neighbouring USArray stations that recorded the $M_w = 7.5$ Sumatra event. The 30 stations divide into 5 parallel lines {J, L, M, O, R}, each at constant latitude and featuring a station spacing of $\approx 70$ km (Fig. 6a). Most wave paths yielded sufficiently high cross-correlation coefficients for $\Delta T$ results to be considered robust in at least the lower six frequency bands, from $T_j = 30.0$ s to $5.3$ s period—Fig. 6(b).

The traveltime anomalies $\Delta T$ in Fig. 6(c) vary significantly across neighbouring stations—this could be mantle, crustal, or near-source effects. But the $\Delta T$ vary almost equally strongly with frequency—a colour change within a row indicates frequency dispersion and hence laterally heterogeneous 3-D structure somewhere on the wave path (rather than diffractive dispersion from the spherically symmetric core). 30 s to 7.5 s is a frequency window where dispersion is not expected to be dominated by reverberations in sediments (Zhou et al. 2003) or finite-frequency effects of heterogeneous crust (Ritsma et al. 2009) and should thus represent ‘useful’ signal for mantle tomography. Frequency dependence of $\Delta T$ due to wave propagation in the crust is not modelled because we apply ray-theoretical crustal corrections, but the expected error is small. In continental crust of 35–45 km thickness beneath the stations in Fig. 6 (Bassin et al. 2000), the difference between ray-theoretical and true traveltime in the crust is predicted to be less than 0.1 s for the short wave periods ($T < 30$ s) used here (Ritsma et al. 2009). Unmodelled $\Delta T$ variations across neighbouring stations, which sit on similar crust, would be even smaller.

The observed frequency dependence of Pdiff traveltimes in Fig. 6 is relatively strong compared to the typical 0.1–0.3 s observed for teleseismic P waves in this period range (Sigloch & Nolet 2006). This suggests a significant contribution from heterogeneities in the lowermost mantle. To investigate further, Fig. 7 compares frequency-dependent $\Delta T$ recordings on USArray from two earthquakes at almost identical backazimuths: Event 1 at core-gridded distance, Event 2 at teleseismic distance. Pdiff traveltime patterns in Fig. 7(b) resemble teleseismic traveltime patterns in Fig. 7(c), suggesting an origin mostly in the upper(most) mantle and crust, which are sampled in similar geometries by both wave types.

Stationwise subtraction of $\Delta T$ of Event 2 from Event 1 (Pdiff minus P) should thus enhance the non-upper mantle contribution. Indeed the differential $\Delta T$ pattern in Fig. 7(d) is quite different. The systematic north–south gradient (green to red), more or less parallel to the event backazimuth and ‘banded’ with epicentral distance, which may indicate that our spherically symmetric reference model needs adjustment, or may be a signature of the elliptic earth (the $\Delta T$ have been corrected for ellipticity, though technically not completely correct for core-grazing waves (Kennett & Gudmundsson 1996)). However, the pattern in Fig. 7(d) is not exclusively banded
Multifrequency measurements of $P_{\text{diff}}$

Figure 6. Example traveltime dispersion patterns for $P_{\text{diff}}$ waves, observed on a cluster of neighbouring USArray stations for the Sumatra event of Fig. 1. (a) 30 spatially neighbouring USArray stations grouped as five linear sub-arrays of constant latitudes J, L, M, O, R. (b) Each pixel shows the cross-correlation coefficient $\chi_c$ (in colour) for one finite-frequency measurement. Stations in the same sub-array are grouped in neighbouring matrix rows. Most stations (30 rows) and frequency bands (8 columns) yielded measurements of sufficiently high quality ($\chi_c \geq 0.8$) for an interpretation of traveltime anomalies to be meaningful. The centre periods of frequency bands 1 to 8 are (in seconds): 30.0, 21.2, 15.0, 10.6, 7.5, 5.3, 3.7, 2.7. (c) Traveltime delays $dT$ in seconds. A change of colour within a row indicates a frequency-dispersive wave path. Patterns that change from row to row cannot exclusively reflect a biased reference model and must be partly due to 3-D heterogeneity somewhere along the wave path. A lowermost mantle contribution is suspected because the dispersion here exceeds the 0.1–0.3 s typically observed for teleseismic $P$ waves by Sigloch & Nolet (2006). However, the columnwise ‘banded’ structure of very large delays in the lowest bands is probably due either to IASP91 being a biased reference model (cf. Fig. 5) or to imperfect ellipticity corrections for the $P_{\text{diff}}$ arrivals.

3D Signature of Lower-Mantle Origin

The last two panels show traveltime dispersion, where $dT$ measured in the 30 s band is subtracted from $dT$ in the 15 s band. Fig. 7(e) shows this for the diffracted data of Event 1, that is, $dT(P_{\text{diff}}; T_d = 30 \text{ s}) - dT(P_{\text{diff}}; T_d = 15 \text{ s})$, and Fig. 7(f) for the teleseismic Event 2, that is, $dT(P; T_d = 30 \text{ s}) - dT(P; T_d = 15 \text{ s})$. Such dispersion arises because wave sensitivity to mantle structure is frequency dependent (the size of the Fresnel zone relative to structural anomalies changes). Again the $dT$ dispersion patterns should be similar if due to upper mantle structure, which is sensed similarly by both wave types, and different if caused by 3-D heterogeneities deeper down. The dispersion observations are seen to resemble each other in some aspects, but substantially differ in others (e.g. red in California for $P_{\text{diff}}$, blue for $P$). Also note the larger magnitude of dispersion (more intense colours) for $P_{\text{diff}}$ than for $P$. Hence, beyond the known, strongly dispersive signature of the upper mantle under USArray (Sigloch & Nolet 2006; Sigloch 2008), there seems to be a clear lower-mantle signature present to be exploited by $P_{\text{diff}}$-tomography.

3 Global Data Set of Multifrequency $P_{\text{diff}}$ and $P$ Traveltime Measurements

We present the global data set of $P$-diffracted and teleseismic $P$-wave $dT$ anomalies assembled so far, and assess the factors that determine measurement success: mainly source–receiver distance, source magnitude and frequency band. Measurement success is defined as the cross-correlation coefficient $\chi_c$ between observed and synthetic waveforms exceeding $\chi_c \geq 0.8$—a heuristic value obtained from earlier multifrequency inversions (Sigloch et al. 2008; Tian et al. 2009).
Figure 7. The signature of the lowermost mantle in traveltime measurements. (a) Comparison of two events at almost identical backazimuths from USArray. Event 1 occurred at core-diffracted distances (Southern Sumatra, Indonesia, 2009/09/30 10:16:09.25, 0.72° S, 99.87° E, 82 km depth, 7.5 $M_w$). Event 2 occurred at teleseismic distances (Hokkaido, Japan region, 2009/06/05 03:30:33.06, 41.82° N, 143.45° E, 28 km depth, 6.4 $M_w$). Panels (b)–(f) plot $dT$ anomalies in seconds as coloured dots (note that the colour scale changes from panel to panel). Stations lacking data for one event or the other are marked in black. Corrections for ellipticity, crust and station elevation were applied. (b) Event 1, Pdiff traveltime anomalies in the lowest frequency band: $dT(P_{\text{diff}}; T_d = 30\, s)$. (c) Event 2, $P$ anomalies in the lowest frequency band: $dT(P; T_d = 30\, s)$. (d) Differential traveltimes in the lowest frequency band: $dT(P_{\text{diff}}; T_d = 30\, s) - dT(P; T_d = 30\, s)$. This should subtract out most upper-mantle contributions. (e) Event 1, traveltime dispersion for Pdiff: $dT(P_{\text{diff}}; T_d = 30\, s) - dT(P_{\text{diff}}; T_d = 15\, s)$. (f) Event 2, traveltime dispersion for $P$: $dT(P; T_d = 30\, s) - dT(P; T_d = 15\, s)$.

Figure 8. Global distribution of the 1857 earthquake sources (blue beachballs) and 4085 broad-band receivers (red triangles) used in this study. Each source and receiver contributed to at least one successful measurement ($x_c \geq 0.8$) in at least one frequency band. For each source, a broad-band source time function was deconvolved from the waveform data.

Fig. 8 shows the 1857 events and 4085 broad-band stations that contributed at least one usable measurement. 418 226 unique source–receiver paths for Pdiff yielded 479 559 successful, band-passed Pdiff traveltime measurements. 613 057 unique paths for teleseismic $P$ yielded 2 306 755 usable $P$-wave traveltimes. Source and receiver coverage reflects our ‘No data left behind’ philosophy, in that we considered every event of magnitude $m_b \geq 5.8$ that occurred between 1999 and 2010—plus many earlier or smaller
events if they occurred in unusual locations. Work is ongoing on the most recent years, as the deconvolution of source time functions still requires human supervision.

The large volumes of broad-band waveform data were retrieved from the IRIS and ORFEUS data management centres using fully automated Python software built for this purpose: ObsPyLoad and its successor obspyDMT, freely available at http://kasra-hosseini.github.io/obspyDMT, last accessed 17 August 2015 (hosted in GitHub, a community-standard repository) and described in Scheingraber et al. (2013). obspyDMT also executed the instrument correction to ground displacement, bandpass-filtering between 0.01 and 3.5 Hz, local archiving and updating of the collection when new waveforms became available on the IRIS and ORFEUS servers.

Green’s functions for the 1857 events, in the broad spectrum of 0.2 mHz to 1 Hz and of 85 min duration, were computed with the Yspec software including attenuation (Al-Attar & Woodhouse 2008). Per event, this took about 20 hr on 24 cores (480 core-hours). These Green’s functions were convolved with source time functions for the 1857 events, obtained by the linearized method of Sigloch & Nolet (2006).

3.1 Measurement success as a function of epicentral distance, source magnitude and frequency band

Fig. 9 shows histograms of the cross-correlations achieved in $P$ versus Pdiff measurements. High cross-correlations are more often achieved in the low-frequency passbands. This is true for both $P$ and Pdiff, but significantly more pronounced for Pdiff, as expected from its frequency-dependent attenuation properties of Fig. 2.

At the chosen quality threshold of $x_c \geq 0.8$, the fewest successful $P$ measurements are made in the highest frequency band, whereas the fewest successful Pdiff measurements are made at dominant periods around 7.5 s, the band of the secondary microseismic noise. This becomes more evident in Fig. 10, which shows the number of successful measurements versus dominant period. For teleseismic $P$, the number increases with dominant period, whereas good Pdiff measurements are least common in the microseismic noise band (shaded yellow) rather than at the shortest periods.

Fig. 11 plots the percentage of successful measurements versus epicentral distance. In the teleseismic range, receiver distance has no significant influence on measurement success. Frequency band has a moderate influence: at the lowest frequencies, about 55 per cent of attempted source-receiver pairs yield a good measurement, but only about 40 per cent in the highest frequency band. By contrast, measurement success in the $P$-diffracted range drops precipitously as receiver distance increases. Frequency band is seen to have a relatively larger influence than for teleseismic $P$: measurement success is 2–3 times better in the lowest bands than in the high or microseismic noise bands. The much lower success with distance mirrors the exponential drop of Pdiff wave amplitude with distance in Fig. 2. Since the level of random noise in a recording is largely independent of epicentral distance, signal-to-noise ratio decreases as distance increases, and observed waveforms tend to become dominated by noise, no longer fitting the synthetics. This also explains the minimum of usable measurements in the microseismic band, where absolute noise levels exceed those in higher or lower frequency bands, so that signal-to-noise ratio drops below usable levels sooner as a function of distance.

Large earthquakes generate large wave amplitudes, and hence we expect a strong positive effect of earthquake magnitude on measurement success for Pdiff—stronger than for teleseismic $P$. Fig. 12 confirms this. Percentage of successful measurements is plotted against source magnitude, separately for receivers in the close ($32^\circ–60^\circ$) and far ($60^\circ–95^\circ$) teleseismic ranges, and in the close ($95^\circ–120^\circ$) and far ($>120^\circ$) Pdiff ranges. The results for the two teleseismic groups are very similar: about 30–40 per cent of source-receiver pairs yield successful measurements for the weakest events of $M_w = 5.5$ (percentage averaged over all frequency bands). Success rates decrease with event size, but level off at 60 per cent for $M_w = 6.5$, and even drop again for the largest events (because their true source time functions are more complex than accommodated by our simple point source estimates).

The systematics are different for Pdiff: below magnitude $M_w < 6$, only a small fraction of measurements are successful, especially in the far teleseismic range. Measurement success increases strongly with event magnitude, again confirming that Pdiff measurements are limited mainly by signal-to-noise ratio. Magnitude 8 events produce remarkable success ratios of $>35$ per cent even in the far teleseismic range, which equals the success ratios for teleseismic $P$. In order to achieve $>35$ per cent good measurements, event magnitudes $M_w$...
Figure 10. Histograms of satisfactory $P$ and $P$diff measurements, as a function of frequency band. A measurement is included only if it achieved cross-correlation coefficient $\chi_c \geq 0.8$. Yellow shading marks the secondary microseismic noise band. Top: for teleseismic $P$ waves, measurement success increases (almost) monotonously with wave period. Bottom: for $P$diff waves, measurement success is lowest in the band of the secondary microseismic noise.

must be approximately 5.5, 5.7, 6.5 and 8.0 for the near-$P$, far-$P$, near-$P$diff and far-$P$diff ranges, respectively.

Fig. 13 shows histograms of measured traveltime anomalies $dT$ in the $P$ versus $P$diff ranges, sorted by frequency band (colours). The $dT$ are corrected for the earth’s ellipticity, for station topography, and for crustal structure using crustal model CRUST2.0 (Bassin et al. 2000). Standard deviations range between 1.44 s and 2.14 s for teleseismic $dT$ (depending on frequency band), and between 1.92 s and 3.94 s for $P$-diffracted $dT$. The relative differences between frequency bands and distance ranges are real, but the absolute $dT$ values are conservative estimates because the mean of the $dT$ has been removed eventwise. A non-zero mean could result from a mis-estimated origin time rather than from 3-D structural heterogeneity, the signal of interest. On the other hand, removing the mean may discard some signal that is truly due to earth structure, hence ‘conservative’. The mean was not calculated on all measurements—this would yield decidedly non-Gaussian histograms because the mean would be skewed by a few spatially concentrated station clusters, mainly USAArray. Instead it was calculated on the rather evenly distributed Global Seismographic Network in the lowest frequency band, and subsequently removed from the $dT$ values of all stations and all bands, yielding the largely Gaussian histograms of Fig. 13.

Figure 11. Percentage of successful $dT$ measurements ($\chi_c \geq 0.8$), as a function of epicentral distance and frequency band. In the teleseismic $P$ range, measurement success depends more on the frequency band than on epicentral distance, and varies between 55 and 40 per cent in the lowest and highest bands, respectively. In the $P$-diffracted range, measurement success rate drops sharply with increasing distance, mirroring the exponential drop in signal amplitudes of Fig. 2. Success rate is lowest in the secondary microseismic noise band (yellow and pink lines, consistent with Fig. 10).

Figure 12. Percentage of successful $dT$ measurements ($\chi_c \geq 0.8$), as a function of earthquake source magnitude $M_w$ and epicentral distance range. In the core-diffracted range (red and purple lines), the rate of successful measurements grows with earthquake magnitude. In the teleseismic $P$ range (green and blue), this is only true for small magnitudes ($M_w < 6.5$). For larger events the success rate drops again, because our point-source approximation becomes inadequate.

3.2 Global mantle heterogeneity inferred from $P$diff traveltime anomalies

We turn to the structural information contained in our large, global set of $P$-diffracted traveltime data. Out of 418 226 candidate $P$diff source–receiver paths, 165 651 yielded at least one successful passband measurement. 3 345 808 attempts at measurements (418 226 paths times 8 frequency bands) yielded 479 559 multifrequency $dT$ values of $\chi_c \geq 0.8$. The $dT$s are corrected for the effects of ellipticity, crust and station elevation.

Fig. 14 plots the coverage of the CMB by nominally core-grazing segments of 479 559 $P$diff paths that yielded successful measurements. Showing the Fresnel zones of finite-frequency kernels instead of rays would be a more faithful approximation of true measurement sensitivities near the CMB, but this major computational effort was not complete at the time of writing. The CMB was subdivided into blocks of 0.5°, that is, $720 \times 720$ blocks, and for each block, the lengths of all core-grazing segments visiting it are
corrections were applied. Only measurements that yielded mean of \( dT \) paths that originate from seismicity in South Sandwich). Fig.14 also example located under South America and the adjacent Pacific (from USArray appears as localized, red–yellow patches, with the clearest under the oceans than under the continents. The heavy footprint of sampled. Overall, however, the lowermost mantle is better sampled hemisphere, especially the southern Atlantic, are the most poorly are most densely sampled (dark red). The oceans of the southern mantle under the northern Pacific and East Asia are most densely sampled (dark red). The oceans of the southern hemisphere, especially the southern Atlantic, are the most poorly sampled. Overall, however, the lowermost mantle is better sampled under the oceans than under the continents. The heavy footprint of USArray appears as localized, red–yellow patches, with the clearest example located under South America and the adjacent Pacific (from paths that originate from seismicity in South Sandwich). Fig. 14 also permits to gauge the extent to which wave paths cross, an important proxy for tomographic resolution.

Fig. 15 shows the same ray coverage as Fig. 14, but projected on the CMB are traveltime anomalies \( dT \), permitting to assess the spatial distribution of seismically fast and slow regions in the lowermost mantle. This rough ‘proto-tomography’ exercise assumes that all arrival delays or advances originate from the core-grazing path segments. Each ray segment is coloured by the relative traveltime delay incurred w.r.t. the reference velocity model IASP91 and the Q values of PREM. Red paths are seismically slow, blue paths are fast. Anomalies are seen to be on the order of 1 per cent.

Large-scale structures emerge that are well-known from global tomography models, such as generally slow regions beneath the southern hemisphere, corresponding to the African and Pacific Large Low Shear Velocity Provinces (LLSVPs). Seismically fast areas are confirmed under most of eastern Asia and under Central America, presumably the resting places of massive accumulations of subducted lithosphere. A striking east-west velocity gradient is evident across the USArray-sampled patch beneath South America and adjacent Pacific (compare to Fig. 14). Other strong velocity gradients appear to rim the Pacific LLSVP.

4 DISCUSSION

For waveform measurements on Pdiff waves, the main challenge is their signal-to-noise ratio. Wave amplitudes that rapidly decline with epicentral distance and with frequency (Figs 2 and 11) meet with ambient noise levels that peak in the microseismic frequency band. Our approach mitigates by attempting measurements in 8 different passbands across the broad-band range of 30.0–2.7 s period, maximising the chance that at least one measurement will succeed, so that the wave path is basically covered. Fitting a passband waveform is easier than a broad-band measurement, since the latter depends on good SNR across all frequencies. This is particular pertinent in selectively high-noise environments, as for Pdiff waves or on the ocean floor (where signal is not weak per se, but microseismic noise is elevated compared to land stations).

Over all eight bands and all distances, 14.3 per cent (479 559) high-quality, multifrequency Pdiff measurements were achieved, compared to 47.0 per cent (2 306 755) teleseismic P measurements. Diffracted P-waves measurements are most likely to succeed in the lowest frequency band of 30.0 s (21.5 per cent success rate) and least likely in the 7.5 s period band of the microseismic noise (9.0 per cent). Successful teleseismic P measurements are most likely at 21.2 s (53.8 per cent) and least likely in the highest band of 2.7 s (38.8 per cent).

For a given wave path, the frequency dispersion between \( dT \) at the low and high ends of our frequency range can be 1–2 s, although more of this seems to be due to a mismatch of the spherically symmetric reference model rather than to 3-D heterogeneity (Fig. 6). Nevertheless, the signature of 3-D mantle heterogeneity is clearly present in Figs 6 and 7. A substantial part of this signal seems to originate from the lowermost mantle, since \( dT \) dispersion is significantly stronger for Pdiff than for P.

Computing Green’s functions and sensitivity kernels up to the high frequencies used here is challenging and currently only feasible in a spherically symmetric reference model. This limitation may not be a serious one, given that mantle heterogeneity is expected to be of only a few percent, except perhaps in Ultra-Low Velocity Zones (ULVZs), which are however very limited spatially. Hence a single-iteration matrix inversion, starting from a spherically symmetric reference model, should be capable of yielding a very good approximation to true earth structure. We hope and expect that our ability to exploit the highly resolving short wavelengths will far outweigh the limitation of not being able to iteratively update the solution.

The Yspec and AxiSEM software packages tools are both capable of delivering accurate Green’s functions in spherically symmetric reference models, for the ambitious, broad-band frequency range in this study. Since AxiSEM was not complete at the time of creating our synthetic archive, Yspec was used to calculate all the traces in \( \approx 2000 \) events of our data set which cost \( \approx 10^6 \) CPU hours in total. We intend to adapt the work flow to incorporate the Instaseis database approach of van Driel et al. (2015). This expends a large upfront effort on computing and storing generic global Green’s functions componentwise using databases generated by AxiSEM. Extraction from the database and linear combination into specific Green’s functions then becomes quasi instantaneous (on the order of milliseconds). Sensitivity kernels from full forward and backward
Figure 14. Sampling of the lowermost mantle by our data set of 479,559 Pdiff traveltime anomalies. The same data are shown in two different map views, Atlantic-centred at the top, Pacific-centred at the bottom. Each line represents the core-grazing segment of a ray-theoretical Pdiff path that yielded a successful measurement (i.e. $x_c \geq 0.8$ in any frequency band). Density of ray path coverage is indicated by colours (note that the colour scale is logarithmic). The CMB under oceans tends to be better sampled than under continents, especially in the southern hemisphere. The true sensitivity of a Pdiff wave to core–mantle boundary structure is ellipsoidal rather than ray-like—covering a broader area of the CMB and extending hundreds of kilometres into the overlying mantle.

wavefields will also be computed with AxiSEM, since Yspec is not an efficient tool for this purpose.

Deconvolution of source parameters is the bottleneck that requires the most user supervision, and no earthquake centre currently does this work. Stähler & Sigloch (2014) demonstrated a scheme of probabilistic inference that should largely automate this step. It also delivers full source uncertainty estimates and propagates them into the (correlated) uncertainties of $dT$ measurements, as required to properly fill the measurement covariance matrix for tomography.

In terms of structural heterogeneity at long spatial wavelengths, the results of Fig. 15 are encouragingly consistent with prior studies that used different methods and data. This includes the aforementioned LLSVPs and the slab graveyards under Eastern Asia and Central America, which have become robust features in global tomographic models, especially S-wave models, such as S362ANI (Kustowski et al. 2008), S40RTS (Ritsema et al. 2011), SAW24B16 (Mégnin & Romanowicz 2000), IMSL-S (Houser et al. 2008), or GyPSuM (Simmons et al. 2010). Because P-wave models do not have the benefit of including normal modes, their resolution of lowermost mantle structure is generally considered less reliable and more attenuated. While too early to assess the improvements possible through Pdiff in this regard, the large number of successful Pdiff measurements and their general agreement with known structure is promising.

Since the pioneering studies of Wysession (1996) and Kárason & Van der Hilst (2001), Pdiff waves have not been explicitly included in global P-wave tomographies. Both studies used a data set of 543 PKPdf - Pdiff differential traveltimes. By comparison, we obtained 479,559 usable Pdiff waveform measurements, demonstrating the sea change in data availability that has taken place. While our structural findings bear more overall resemblance to recent global S-wave models, they are also broadly consistent with these two early, global Pdiff inversions, which recovered the two LLSVPs as well as high-velocity anomalies beneath south/central America and eastern Asia. Not present in our $dT$ map is a high velocity structure under south and southeast of Africa that was apparently required by their Pdiff data in both the Wysession (1996) and the Kárason & Van der Hilst (2001) study.

From Pdiff measurements made by Ritsema & van Heijst (2002), Koelemeijer et al. (2013) constructed a traveltine map similar to our Fig. 15, by projecting the entire $dT$ anomalies onto the CMB. With a coarse gridding of $5^\circ$ caps, only large-scale structure was targeted.
Multifrequency measurements of $P_{\text{diff}}$

Figure 15. Traveltime anomalies of 479,559 $P_{\text{diff}}$ waves, projected onto their CMB-grazing segments. As Fig. 14, except that colour represents the traveltime anomaly $dT$ in per cent. Red designates a measurement delayed relative to IASP91, blue means advanced. For a given delay $dT$, a longer core-grazing segment would result in a fainter shade of red, because $dT$ is spread evenly along the segment. The raw $dT$ measurements were corrected for earth’s ellipticity, station topography and near-station crustal structure. Assuming that most $P_{\text{diff}}$ traveltime anomalies originate in the lowermost mantle, this plot represents a rough ‘proto-tomography’, charting out seismically fast (blue) and slow (red) structural anomalies near the CMB.

They recovered the two LLSVPs and high-velocity anomalies under central America and east Asia—another confirmation of these most robust features in the lowermost mantle.

A relatively small-scale but pronounced feature of Fig. 15 is an intensely slow patch just west of Hawaii. This structure was recently described as a newly recognized ULVZ by Cottaar & Romanowicz (2012), who modelled $S_{\text{diff}}$ waveforms. It appears to be highly visible to $P$ waves as well.

5 CONCLUSION

We presented a method to routinely measure and model core-diffracted $P$ waves across the broad-band frequency range. Green’s functions are calculated by semi-analytical (i.e. theoretically exact) wave propagation through a spherically symmetric reference model, to 1 Hz dominant frequency, and broad-band source time functions are deconvolved from the waveform data. Largely automated processing of 1857 events yielded 2,306,755 teleseismic $P$- and 479,559 $P_{\text{diff}}$-diffracted traveltime measurements by cross-correlation across eight frequency bands (dominant periods 30–2.7 s).

While significantly more challenging than teleseismic $P$-measurements, we obtain very decent success rates for $dT$ measurements on $P_{\text{diff}}$ waveforms, of around 14 per cent across all attempted wave paths and frequencies. The main challenge is signal-to-noise ratio, which drops precipitously with epicentral distance and with frequency, as expected for diffracted waves. $dT$ anomalies are larger for $P_{\text{diff}}$ than for $P$ waves, and frequency dependence of $dT$ due to 3-D heterogeneity is stronger for $P_{\text{diff}}$ as well. Projected on their core-grazing ray segments, the $dT$ measurements recover major structural, lower-mantle heterogeneities known from existing global mantle models based on $P$- and $S$ waves. This methodically novel and very large data set of core-diffracted $P$ waves is ready for global waveform tomography.

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